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Study of Effects of Radiation on Heat Transfer of Two Phase Boundary Layer Flow over a Stretching Sheet

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ABSTRACT: The behaviour of incompressible, laminar, boundary-layer two phase flows over a semi infinite flat plate is considered. The two phase flow problem with suspension is modelled and solved in accordance with two-way coupling model. We have considered the slip-energy term in the particle phase temperature equation. The momentum conservation equation of particle phase, in the vertical direction is considered. The transfer of heat due to conduction and generation of heat from viscous dissipation play a vital role in analysis of boundary layer characteristics of particle phase of two phase flow. So the impact of radiation and particle volume fraction on energy transfer and other features of flow problem have been analyzed. The significant results are presented through graphs and table. The effect of radiation parameter 'Ra' has direct impact on temperature profile of fluid and particle phases.The rise of radiation parameter 'Ra' causes the rising of the temperature of fluid and particle phases . The temperature of fluid phase decreases when it close to the stretching sheet and increases as it away from sheet with increasing value of ' Pr'.

Keywords: Two phase flow, Radiation, Laminar flow.

I. INTRODUCTION

The analysis of the boundary layer flow concept over a movable stretched surface was initially done by sakiadis [16] in 1961. Tsou *et al*., [24] analyzed the procedure of thermal energy transfer theoretically and established the computational result of Sakiadis experimentally. The velocity and temperature distribution of the boundary layer flow on a movable surface are very much essential for their use in diversified field such as the aerodynamic industry, glass blowing, drying of thin sheet, drawing plastic films and etc. This motivated the researchers for more advance study. Grubka *et al*., [13] has worked and computed for the distribution of temperature in the flow over a stretchable sheet. Here we assumed that heat flux is uniform. Anderson *et al*., [1, 2] obtained a different type of similarity solution for the heat energy distribution. Sharidan [17] offered similarity solutions for velocity and temperature distribution in unsteady boundary layer that was generated due to stretching sheet. Aziz [3] has obtained numerical solution of thermal boundary layer of laminar flow over a flat plate which is convective in nature. Chakrabarti [4] have considered the hydro magnetic flow and its effect on heat transfer over a stretchable sheet. Crane [6] has derived the procedure for planar viscous flow over linear stretchable sheet and got in exponential form . Gireesha *et al*., [7-12] have calculated the impact of hydrodynamic laminar flow and heat Transfer of a two phase fluid over an unsteady stretchable surface. They have verified the heat transfer quality for two types of boundary conditions, i.e variable wall temperature and variable Heat flux. Ramesh *et al*., [14-15] have analysed the velocity and temperature distribution properties of hydrodynamic dusty fluid over an inclined stretchable sheet with non uniform heat source/sink. Gireesha *et al.,* [07-12] also analysed the mixed convective flow of a dusty fluid over a stretchable sheet. The study of two phase flows is very much essential, due to it's application in different fields, like analysis environmental pollutant, separation of particles by centrifugal force, blood rheology, etc. In this paper, the behaviour of incompressible, laminar boundary-layer flows of gas with particles over a semi infinite flat plate along the full length of the plate is analyzed. The solution of two phase flow problems is handled in the way of two-fluid approach.

In this paper, the terms connected with the heat supplemented to the system to slip-energy flux in the temperature equation of particle phase is considered Soo [18-23]. The momentum equation for particle phase in vertical direction is not neglected where as neglected in fluid phase. The change of temperature due to conduction and viscous dissipation in the heat equation of the particulate phase has been accepted for better results of the boundary layer components. The role of volume fraction and other boundary layer components on heat transfer have been analyzed.

II. MATHEMATICAL FORMULATION

Let us think a two dimensional boundary layer flow of viscous fluid containing dust; over a vertical stretchable sheet. The flow characteristics are steady, laminar and incompressible. The flow is formed by the action of two equal and opposite forces along the horizontal direction. Y-axis is the normal to the flow. As the force is applied in horizontal direction and by virtue of properties of stretching sheet, it expanded with the velocity $U_w(x)$ in the direction of x-axis, being the origin fixed in the fluid. The temperature of fluid is T at the beginning i.e before the flow. It is assumed that the fluid-particle mixing is uniform throughout the fluid.

The mathematical equations representing above problem are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial}{\partial x}(\rho_p u_p) + \frac{\partial}{\partial y}(\rho_p v_p) = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho}\,\mu\frac{\partial^2 u}{\partial y^2} - \frac{1}{(1-\varphi)\rho}\frac{1}{\tau_p}\varphi\rho_s\,(u-u_p) + g\beta^*\,(T-T_\infty) + \frac{1}{1-\varphi}\frac{\rho_p}{\rho}\left(\frac{e}{m}\right)E \quad (3)$$

$$\varphi\rho_s\,(u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y}) = \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial u_p}{\partial y}\right) + \frac{1}{\tau_p}\varphi\rho_s\,(u-u_p) + \varphi(\rho_s - \rho)g + \rho_p\left(\frac{e}{m}\right)E \quad (4)$$

$$\varphi\rho_s\,\left(u_p\frac{\partial v_p}{\partial x} + v_p\frac{\partial v_p}{\partial y}\right) = \frac{\partial}{\partial y}\left(\varphi\mu_s\frac{\partial v_p}{\partial y}\right) + \frac{1}{\tau_p}\varphi\rho_s\,(v-v_p) \quad (5)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\varphi\rho_s c_s}{(1-\varphi)c_p}\frac{1}{\rho\,\tau_T}(T_p - T) + \frac{\varphi\rho_s}{(1-\varphi)}\frac{1}{\rho c_p}\frac{1}{\tau_p}(u_p-u)^2 + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{q'''}{\rho c_p} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{1}{1-\varphi}\frac{1}{\rho c_p}\rho_p\left(\frac{e}{m}\right)Eu_p \quad (6)$$

$$u_p\frac{\partial T_p}{\partial x} + v_p\frac{\partial T_p}{\partial y} = -\frac{1}{\tau_T}(T_p - T) + \frac{1}{\varphi\rho_s c_s}\frac{\partial}{\partial y}\left(\varphi k_s\frac{\partial T_p}{\partial y}\right) - \frac{1}{\tau_p}\frac{1}{c_s}(u-u_p)^2 + \frac{\mu_s}{\rho_s c_s}\left[u_p\frac{\partial^2 u_p}{\partial y^2} + \left(\frac{\partial u_p}{\partial y}\right)^2\right] + \frac{1}{c_s}\left(\frac{e}{m}\right)Eu_p - \frac{1}{\rho_s c_s}\frac{\partial q_{rp}}{\partial y} + \frac{1}{\rho_s c_s}q_p''' \quad (7)$$

With boundary conditions

$$u = U_w(x) = cx\,, v = 0 \text{ at } y = 0$$
$$\rho_p = \omega\rho, u = 0\,, u_p = 0\,, v_p \to v \text{ as } y \to \infty \quad (8)$$

Where ω is the ratio between particles and fluid in the main stream.

III. SOLUTION OF THE PROBLEM

The Eqns. (1) to (7) are condensed to a system of ODEs by using Shooting method and then solved them applying well known RKF-45 method.

In order to solve Eqns. (6) and (7), the non-dimensional temperature boundary conditions are as follows

$$T = T_w = T_\infty + A\left(\frac{x}{l}\right)^2 \text{ at } y = 0$$
$$T \to T_\infty\,, T_p \to T_\infty \text{ as } y \to \infty$$

Where A is a positive constant, $l = \sqrt{\frac{\nu}{c}}$.

For most of the gases $\tau_p \approx \tau_T$,

$k_s = k\frac{c_s}{c_p}\frac{\mu_s}{\mu}$ if $\frac{c_s}{c_p} = \frac{2}{3P_r}$

Eqns. (1)–(7) can be non dimenssionlized as follow

$$u = cxf'(\eta)\,, v = -\sqrt{c\nu}f(\eta)\,, \eta = \sqrt{\frac{c}{\nu}}\,y\,, u_p = cxF(\eta)\,, v_p = \sqrt{c\nu}\,G(\eta)\,,$$

$$\rho_r = H(\eta),\ \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}$$

$\theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}$, where $T - T_\infty = A\left(\frac{x}{l}\right)^2\theta\,, T_p - T_\infty = A\left(\frac{x}{l}\right)^2\theta_p$

$\beta = \frac{1}{c\tau_p}$, $\epsilon = \frac{\nu_s}{\nu}$, $P_r = \frac{\mu c_p}{k}$, $E_c = \frac{c^2 l^2}{Ac_p} = \frac{\nu c}{Ac_p}$, $F_r = \frac{c^2 x}{g}$, $\gamma = \frac{\rho_s}{\rho}$, $G_r = \frac{g\beta^*(T-T_\infty)}{c^2 x}$, $\nu = \frac{\mu}{\rho}$, $\rho_r = \frac{\rho_p}{\rho}$, $M = \left(\frac{e}{m}\right)\frac{E}{c^2 x}$, $R_a = \frac{16T_\infty^3\sigma^*}{3k^*k}$

c is the stretching rate

β is the fluid particle interaction parameter. Other symbols have usual meaning.

We get

$$H' = -\frac{(HF + HG')}{G} \quad (9)$$

$$f'''(\eta) = \left(f'(\eta)\right)^2 - f(\eta)f''(\eta) - \frac{1}{(1-\varphi)}\beta\,H(\eta) - \mathrm{Gr}\theta - \frac{\mathrm{M}}{1-\varphi}\mathrm{H} \quad (10)$$

$$F''(\eta) = \left[G(\eta)F'(\eta) + [F(\eta)]^2 - \beta\left[f'(\eta) - F(\eta)\right] - \frac{1}{Fr}\left(1 - \frac{1}{\gamma}\right) - M\right]/\epsilon \quad (11)$$

$$G''(\eta) = \left[GG' + \beta\,(f + G)\ \right]/\epsilon \quad (12)$$

$$\theta'' = \left(Pr\left(2f'\theta - f\theta'\right) - \frac{2}{3}\frac{\beta}{1-\varphi}H\left[\theta_p - \theta\right] - \frac{1}{1-\varphi}PrE_c\beta H\left[F - f'\right]^2 - \mathrm{PrE_c f''^2} - \lambda\theta - \frac{1}{(1-\varphi)}H(\eta)ME_c\,Pr\,F(\eta)\right)/(R_a + 1) \quad (13)$$

$$\theta_p''(\eta) = (2F\theta_p + G\theta_p' + \beta\left[\theta_p - \theta\right] + 1.5\beta EcPr\left[f' - F\right]^2 - \frac{3}{2}\epsilon EcPr\left[FF'' + (F')^2\ \right] - \frac{3}{2}MEcPrF(\eta) - \lambda_p\theta_p)/\left(\frac{\epsilon}{Pr} + \frac{3}{2}\frac{R_a}{\gamma}\right) \quad (14)$$

with boundary conditions

$$G'(\eta) = 0, f(\eta) = 0, f'(\eta) = 1, F'(\eta) = 0,$$
$$\theta(\eta) = 1\,, \theta_p' = 0 \text{ as } \eta \to 0`$$
$$f'(\eta) = 0, F(\eta) = 0, G(\eta) = -f(\eta)\,,$$
$$H(\eta) = \omega\,, \theta(\eta) \to 0\,,\quad \theta_p(\eta) \to 0 \text{ as } \eta \to \infty \quad (15)$$

Here in this problem the value of $f''(0)$, $F(0), G(0), H(0), \theta'(0), \theta_p(0)$ are not known but $f'(\infty) = 0, F(\infty) = 0, G(\infty) = -f(\infty), H(\infty) = \omega, \theta(\infty) = 0$, $\theta_p(\infty) = 0$ are given. By using Shooting method the value of $f''(0), F(0), G(0)$, $H(0), \theta'(0)$, $\theta_p(0)$ can be obtained. We have supplied $f''(0) = \propto_0$ and $f''(0) = \propto_1$. The improved value of $f''(0) = \propto_2$ is calculated by using interpolation formula of 1st degree. Then the value of $f'(\propto_2, \infty)$ is calculated by using Runge-Kutta method. If $f'(\propto_2, \infty)$ is equal to $f'(\infty)$ up to a desired decimal accuracy, then $\propto_2$ i.e $f''(0)$ is determined, otherwise the above procedure is repeated with $\propto_0 = \propto_1$ and $\propto_1 = \propto_2$ until a correct $\propto_2$ is obtained. Similarly the correct values of $F(0), G(0), H(0), \theta'(0), (0)$ can be obtained.

IV. GRAPHICAL REPRESENTATION

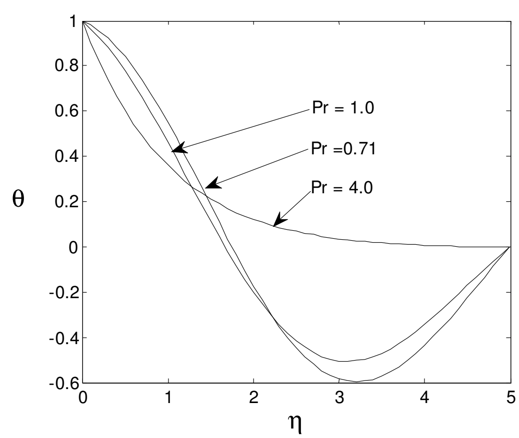

Fig. 1. Effect of Pr on temperture profies of carrier fluid phase for $Ec = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01$, $\gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

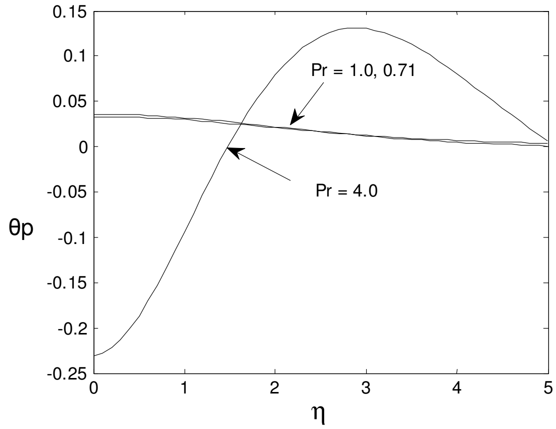

Fig. 2. Effect of Pr on Temperture profies of particle phase for $Ec = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

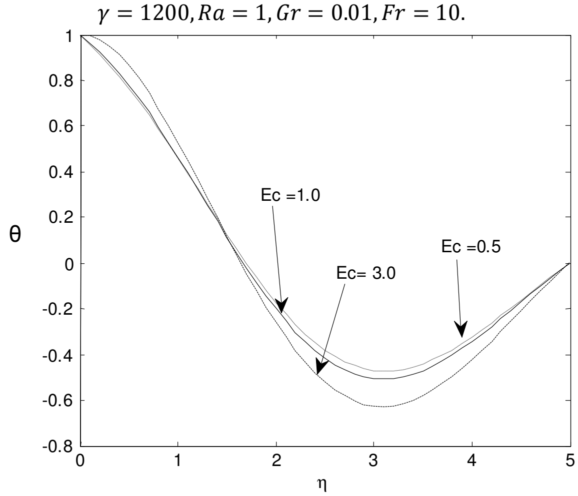

Fig. 3. Impact of Ec on Temperture profies of carrier fluid for $Pr = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

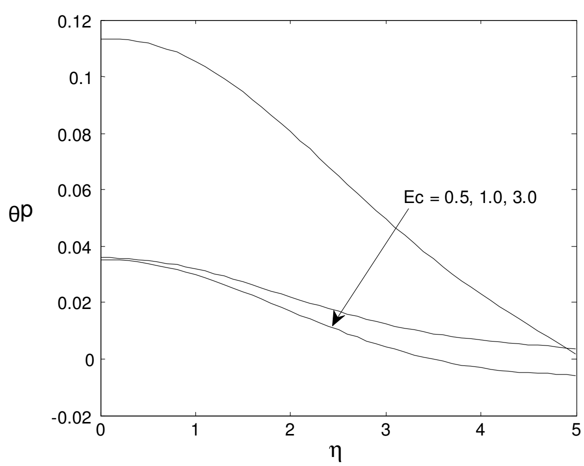

Fig. 4. Effect of Ec on Temperture profies of particle phase for $Ec = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

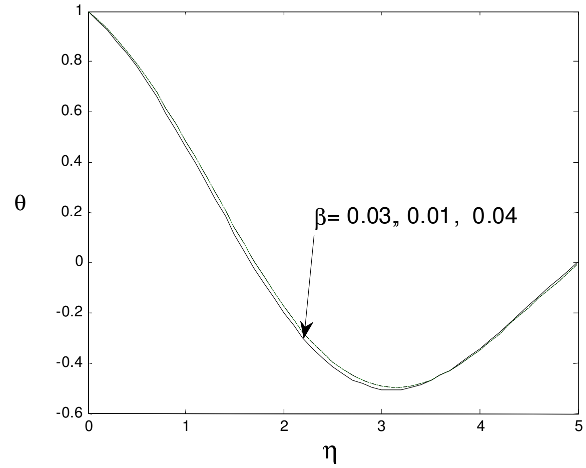

Fig. 5. Effect of β on temperature profies of carrier fluid phase for $Pr = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

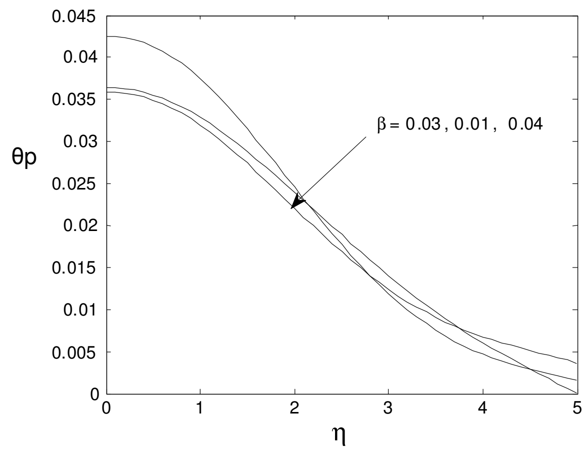

Fig. 6. Effect of β on temperature profies of particle phase for $Pr = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

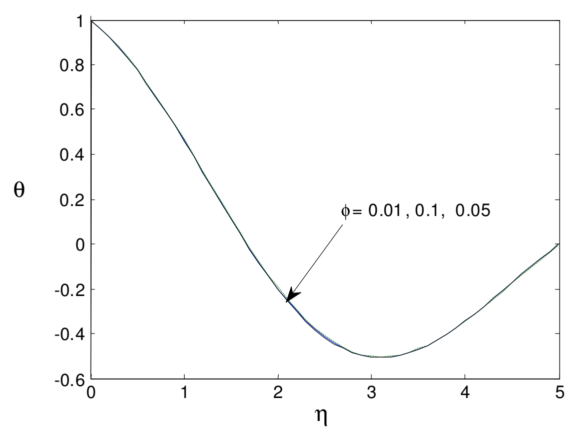

Fig. 7. Impact of φ on temperature profies of carrier fluid for $Pr = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

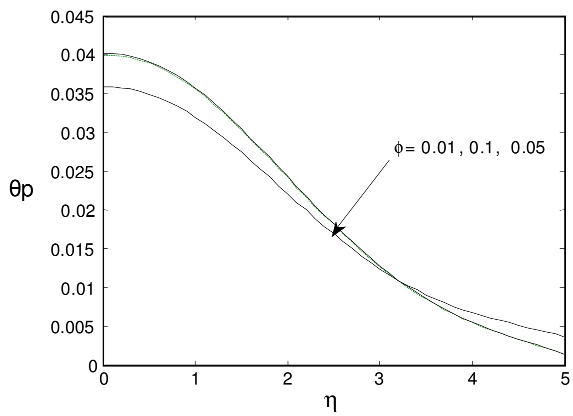

Fig. 8. Impact of φ on temperature profies of particle phase for $Pr = 1.0,\ \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Ra = 1, Gr = 0.01, Fr = 10.$

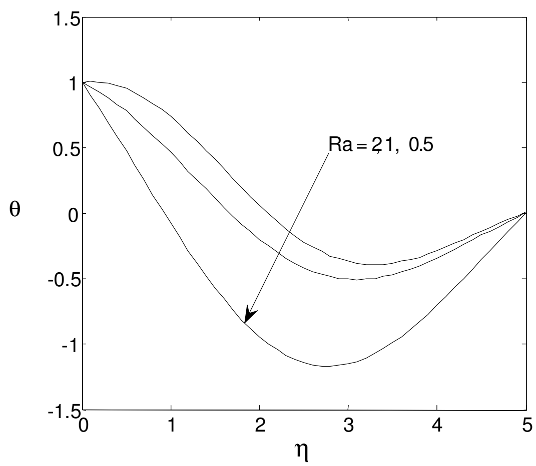

Fig. 9. Impact of Ra upon temperature profies of carrier fluid for $Pr = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Gr = 0.01, Fr = 10.$

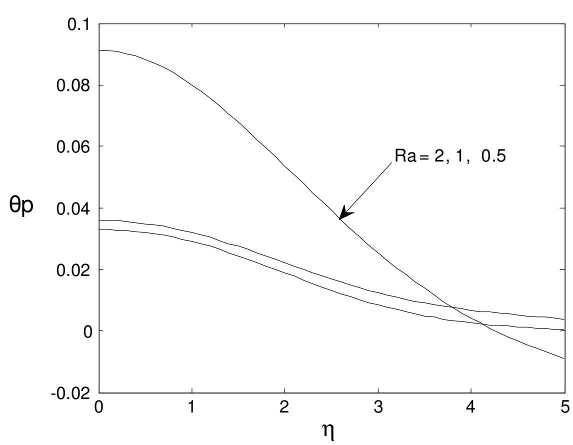

Fig. 10. Impact of Ra upon temperature profies of particle phase for $Pr = 1.0, \epsilon = 5.0, \beta = 0.03, \varphi = 0.01, \gamma = 1200, Gr = 0.01, Fr = 10.$

V. RESULTS AND DISCUSSION

Fig. 1 represents the effect of 'Pr' on temperature outline 'T' of fluid. It is concluded that Pr is inversely proportional with temperature of fluid phase near the sheet but directly proportional at away from the sheet.

Fig. 2 depicts the effect of Pr on temperature profile of the particle phase Tp. It is observed that when Pr increases, the temperature of fluid phase decreases near the plate but increases away from the plate. Fig. 3 illustrates the temperature profile of fluid phase. It is observed that the temperature of fluid phase decreases with the increasing value of 'Ec'. Fig. 4 shows the temperature profile of particle phase. It is noticed that the temperature of particle phase increases with the increasing value of 'Ec'. Fig. 5 describes the effect of 'β', the particle-fluid interaction parameter on fluid phase temperature. The temperature of the fluid phase increases with the increase of 'β'. Fig. 6 explains the effect of 'β' on the particle phase temperature. The temperature of the particle phase increases with the increase of β. Fig. 7 depicts the effect of φ, the volume fraction of particles on fluid phase temperature 'T'. It is observed that no significant effect of φ on fluid phase temperature T. Fig. 8 shows the effect of 'φ' on particle phase temperature Tp. The temperature of the particle phase increases with the increase of 'φ'. Fig. 9 demonstrates the effect of Ra , the radiation parameter on fluid phase temperature 'T'. From this figure it is noticed that the temperature of the fluid phase decreases with the increase of Ra. Fig. 10 represents the effect of 'Ra' on particle phase temperature Tp. From the figure it is concluded that particle phase temperature increases with the increase of radiation 'Ra' .

VI. CONCLUSION

In this chapter the effect of radiation on the two phase fluid over a stretching sheet has been studied. The significant results from the graphs are summarized as follow :

(i) The present study has been done for Fr =10, ϵ = 5.0, γ = 1200, Gr = 0.01.

(ii) The effect of radiation parameter 'Ra' enhances the temperature of both phases.

(iii) The temperature of fluid phase decreases near the stretching sheet and increases away from it with increasing value of ' Pr'.

(iv) The effect of 'φ' has no significant has little impact on particle phase.

CONFLICT OF INTEREST

Authors have no any conflict of interest.

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